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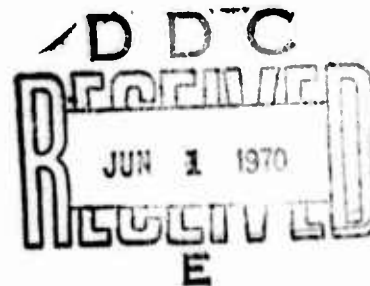
FOREIGN TECHNOLOGY DIVISION



APPRAISING THE RELIABILITY OF A FINITE AUTOMATON,
CONSIDERING ALTERNATING FAILURES

by

G. A. Volovnik



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Block	Italic	Transliteration	Block	Italic	Transliteration
А а	<i>А а</i>	A, a	Р р	<i>Р р</i>	R, r
Б б	<i>Б б</i>	B, b	С с	<i>С с</i>	S, s
В в	<i>В в</i>	V, v	Т т	<i>Т т</i>	T, t
Г г	<i>Г г</i>	G, g	У у	<i>У у</i>	U, u
Д д	<i>Д д</i>	D, d	Ф ф	<i>Ф ф</i>	F, f
Е е	<i>Е е</i>	Ye, ye; E, e*	Х х	<i>Х х</i>	Kh, kh
Ж ж	<i>Ж ж</i>	Zh, zh	Ц ц	<i>Ц ц</i>	Ts, ts
З з	<i>З з</i>	Z, z	Ч ч	<i>Ч ч</i>	Ch, ch
И и	<i>И и</i>	I, i	Ш ш	<i>Ш ш</i>	Sh, sh
Й й	<i>Й й</i>	Y, y	Щ щ	<i>Щ щ</i>	Shch, shch
К к	<i>К к</i>	K, k	Ъ ъ	<i>Ъ ъ</i>	"
Л л	<i>Л л</i>	L, l	Ы ы	<i>Ы ы</i>	Y, y
М м	<i>М м</i>	M, m	Ь ь	<i>Ь ь</i>	'
Н н	<i>Н н</i>	N, n	Э э	<i>Э э</i>	E, e
О о	<i>О о</i>	O, o	Ю ю	<i>Ю ю</i>	Yu, yu
П п	<i>П п</i>	P, p	Я я	<i>Я я</i>	Ya, ya

* ye initially, after vowels, and after ъ, ь; e elsewhere.
 When written as ѣ in Russian, transliterate as yě or ě.
 The use of diacritical marks is preferred, but such marks
 may be omitted when expediency dictates.

APPRAISING THE RELIABILITY OF A FINITE AUTOMATON, CONSIDERING ALTERNATING FAILURES

G. A. Volovnik

A Markovian model has been built and a reliability function of a finite automaton has been obtained to the input of which a random sequence of input signals proceeds. The model reflects the connection of the process of functioning of a finite automaton with the process of structural changes which are caused by possible disturbances. Along with stable disturbances, the disturbances which bear an alternating character are considered.

The tendency to more fully reflect, when calculating reliability, the properties of an actual technical device leads to the necessity of studying the largest possible number of factors which determine reliability.

In this article a model of reliability of a finite automaton is proposed, considering the alternating character of failures and the disturbances of the automaton which are connected with it. Such disturbances, existing for some random time, can be self-eliminating (withdrawing). The moments of appearance of failures are also random.

Formulation of the problem. A finite automaton with the assigned structural layout is under the influence of random input signals, the

law of distribution of which is known. Further, a list is shown of N possible disturbances, characterized by the probabilities of appearance, a_i , and withdrawal b_i , $i = (\overline{1, N})$, which do not depend on the cycle number.

We desire the function of reliability $P(n)$ of the finite automaton for an arbitrary number of cycles n of its work and a number of other reliability characteristics, if the initial state of the automaton is known when $n = 0$. The function of reliability $P(n)$ of a finite automaton is defined as the probability of correct (for n cycles) functioning of the finite automaton under the condition that on each of the n cycles in the automaton there can exist any complex of the considered disturbances [1]. Correct functioning of the finite automaton with disturbances corresponds to the requirement of fulfillment of such transitions, which are accomplished in a completely properly operating automaton with a coinciding sequence of input signals.

In the article we also make the following assumptions:

the work of the automaton is synchronous by cycles, i.e., the input signals are sent in equal time intervals;

the probability distribution of input signals does not depend on the number of cycles;

separate disturbances are independent;

the disturbances do not take the finite automaton beyond the limits of the structural alphabet;

the moments of appearance and withdrawal of disturbances coincide with the moments of reading of cycles.

Under these assumptions, for solving the problem at hand it is possible to use an apparatus of Markov chains. Below, Markov models of the process of structural changes, caused by the appearance and

withdrawal of disturbances, and also process of correct functioning will be obtained. Then a general Markov reliability model of the finite automaton will be built, combining both processes.

The processes of structural changes. For a description of the process of structural changes let us determine the phase space of the states of the finite automaton $E = \{e\}$ by the criterion of the presence of disturbances in it. Let us carry out ordering of all the possible disturbances, allotting to them numbers from one to N . The state can be characterized by the set of numbers of disturbances which are present in the finite automaton, not distinguishing the sequence in which these disturbances appeared. If the state of the finite automaton with a defined complex of s , $s = (\overline{1, N})$ disturbances is designated by $e(i_1 i_2 \dots i_s)$ then in this designation, consequently, any transposition of indices $i_r, i_r = (\overline{1, N})$, which indicate the number of disturbances, is permissible. For a state of total good operating order the designations e_0 are used. Obviously, the total amount of states will be expressed by the number $l = 2^N$.

A change of the state thus introduced, which we will call the structural state of the finite automaton, is caused by the appearance and withdrawal of one or several disturbances in any combination. According to condition, these changes can occur in discrete instants at an interval, equal to cycle duration. In the presence of constant and independent probabilities of the appearance of a_i and withdrawal b_i , $i = (\overline{1, N})$ of disturbances, the sequence of states in time will form a uniform Markov chain. Elements r_{ij} , $i, j = (\overline{1, l})$ of the matrix of probabilities of transition R , for the obtained chain, are determined by the products of characteristics a_i and b_i , pertaining to those disturbances, with which the transition is connected of the finite automaton from structural state e_i to structural state e_j , $e \in E$.

Considering the smallness of values a_1 and b_1 , it is possible with sufficient basis to disregard the possibility of realizing in one cycle the events which are connected with two or more disturbances, as well as with the possibility of appearance and withdrawal of one disturbance in the same cycle. Under such condition matrix R is essentially simplified. As an illustration let us record the matrix of probabilities of transition, describing the process of structural changes in a finite automaton for the case $N = 3$:

$$R = \begin{matrix} & \begin{matrix} e_0 & e_1 & e_2 & e_3 & e_{12} & e_{13} & e_{23} & e_{123} \end{matrix} \\ \begin{matrix} e_0 \\ e_1 \\ e_2 \\ e_3 \\ e_{12} \\ e_{13} \\ e_{23} \\ e_{123} \end{matrix} & \left\| \begin{array}{cccccccc} 1-\delta_0 & a_1 & a_2 & a_3 & 0 & 0 & 0 & 0 \\ b_1 & 1-\delta_1 & 0 & 0 & a_2 & a_3 & 0 & 0 \\ b_2 & 0 & 1-\delta_2 & 0 & a_1 & 0 & a_3 & 0 \\ b_3 & 0 & 0 & 1-\delta_3 & 0 & a_1 & a_2 & 0 \\ 0 & b_2 & b_1 & 0 & 1-\delta_{12} & 0 & 0 & a_3 \\ 0 & b_3 & 0 & b_1 & 0 & 1-\delta_{13} & 0 & a_2 \\ 0 & 0 & b_3 & b_2 & 0 & 0 & 1-\delta_{23} & a_1 \\ 0 & 0 & 0 & 0 & b_3 & b_2 & b_1 & 1-\delta_{123} \end{array} \right\| \end{matrix}, \quad (1')$$

where

$$\delta_0 = \sum_{i_r=1}^3 a_{i_r}; \quad \delta_{i_1} = \sum_{i_r=1}^3 a_{i_r} - a_{i_1} + b_{i_1}, \quad i_1 = 1, 2, 3;$$

$$\delta_{i_1 i_2} = \sum_{i_r=1}^3 b_{i_r} - b_{i_1} + a_{i_2}, \quad i_1, i_2, i_3 = 1, 2, 3; \quad (1'')$$

$$\delta_{123} = \sum_{i_r=1}^3 b_{i_r}.$$

The process of correct functioning of the finite automaton.

Let us assume that a set of signals, determining the functional state of a finite automaton, includes input, internal, and output signals. Then the functioning of a finite automaton can be represented by transitions according to functional states $u \in U$, where U - finite set.

During a random flow of signals at the input, a change of states of a finite automaton in the multitude of functional states U bears

a random character. To each transition of automaton from state u_i to state u_j , the probability q_{ij} can be ascribed, which is established by probabilistic distribution of the input signals. The introduction of a matrix of probabilities of transition $Q = ||q_{ij}||$, $i, j = (\overline{1, k})$, where k is the number of functional states, determines the process of functioning as a uniform Markov chain [2].

The appearance of a disturbance in a finite automaton in general changes its law of functioning. However, in realizing a favorable sequence of input signals, action of these disturbances can appear in an arbitrarily prolonged time, since in a finite automaton the same transitions will occur as in a properly working automaton. The correct functioning of a finite automaton with disturbances can be described by a matrix order k , which is a modification of

matrix Q . We will designate this matrix through $Q^{(i_1 i_2 \dots i_s)} = ||q_{ij}^{(i_1 i_2 \dots i_s)}||$, if in the finite automaton a complex of disturbances of the type $(i_1 i_2 \dots i_s)$ was formed. (For matrix Q , an equivalent designation in this plan will be $Q^{(0)} = ||q_{ij}^{(0)}||$.)

According to the definition of correct functioning of a finite automaton with disturbances, for the formation of matrix

$Q^{(i_1 i_2 \dots i_s)}$ one should make those elements $q_{ij}^{(0)}$ in matrix $Q^{(0)}$, which correspond to the transitions not preserved in an automaton with disturbances, equal to zero. The transitions accomplished in an automaton with disturbances, like in a properly working automaton, have coinciding elements of matrices

$Q^{(i_1 i_2 \dots i_s)}$ and $Q^{(0)}$.¹

¹In source [1], for a description of the functioning of a finite automaton with disturbances a stochastic matrix of order $k + 1$ is used, in which state $(k + 1)$ is defined as a state of failure.

Let us move on directly to the construction of a model of reliability of the finite automaton, considering the interaction of the process of functioning of an automaton with the process of structural changes.

A Markov model of reliability of a finite automaton. Let us introduce the full set of states of finite automaton M as the direct product of the phase space of structural states E and the set of functional states U , i.e., $M = E \times U$, the elements of which are the pairs $m = (e, u)$, where $e \in E$ and $u \in U$. In conformity with the building of phase space E , set M is also split into nonintersecting subsets $M_0 = \{e_0, U\}$ and $M_{i_1 i_2 \dots i_s} = \{e_{i_1 i_2 \dots i_s}, U\}$, $s = \overline{1, N}$.

In examining the rules of correct functioning of a finite automaton according to the states of set M , which are reflected by matrices $Q^{(0)}$ and $Q^{(i_1 i_2 \dots i_s)}$, depending upon the structural state of the automaton, and qualifying the incorrect transitions in the automaton with disturbances as transitions to the state of failure M_{OTH} , we will be convinced that the process, described by a uniform Markov chain in phase space $M' = M + M_{OTH}$, occur. A matrix of probabilities of transition T of the obtained general Markov chain can be shown in the form of a block matrix of the order $l + 1$. For the case $N = 3$, matrix T has this form:

$$T = \begin{array}{c|cccccc} & M_0 & M_1 & M_2 & M_3 & M_{12} & \\ \hline M_0 & (1-\delta_0)Q^{(0)} & a_1 Q^{(0)} & a_2 Q^{(0)} & a_3 Q^{(0)} & 0 & \\ M_1 & b_1 Q^{(1)} & (1-\delta_1)Q^{(1)} & 0 & 0 & a_2 Q^{(1)} & \\ M_2 & b_2 Q^{(2)} & 0 & (1-\delta_2)Q^{(2)} & 0 & a_1 Q^{(2)} & \\ M_3 & b_3 Q^{(3)} & 0 & 0 & (1-\delta_3)Q^{(3)} & 0 & \\ M_{12} & 0 & b_2 Q^{(12)} & b_1 Q^{(12)} & 0 & (1-\delta_{12})Q^{(12)} & \rightarrow \\ M_{13} & 0 & b_3 Q^{(13)} & 0 & b_1 Q^{(13)} & 0 & \\ M_{23} & 0 & 0 & b_3 Q^{(23)} & b_2 Q^{(23)} & 0 & \\ M_{123} & 0 & 0 & 0 & 0 & b_3 Q^{(123)} & \\ M_{OTH} & 0 & 0 & 0 & 0 & 0 & \end{array} \quad (2)$$

where $P_m(0)$ - the vector of initial probability distribution of states; z - the variable of transform; V - the unit matrix; $[T - zV]^{-1}$ - the matrix, inverse to matrix $[T - zV]$.

The function of reliability of a finite automaton, according to formula (3), will be obtained by adding all the components of vector $P_m(n)$, except the last, equal to failure probability. By grouping the components of vector $P_m(n)$ in a certain way, it is possible to separate the individual components of reliability function. Thus, for example, the probability of un failing work of a finite automaton for n cycles in the presence in it of disturbances with numbers i_1, i_2, \dots, i_s , regardless of the order of their appearance, is equal to

$$P(i_1, \dots, i_s)(n) = \sum_{m \in M_{i_1, i_2, \dots, i_s}} P_m(n), \quad (5)$$

and the probability of un failing work of a finite automaton for cycles in the presence in it of any s disturbances

$$P(s)(n) = \sum_{(i_1, i_2, \dots, i_s)} P(i_1, \dots, i_s)(n), \quad (6)$$

where summation is conducted on all sets (i_1, i_2, \dots, i_s) , distinguished by subscripts designating the number of disturbances.

Let us note that in the case of showing the matrix of probabilities of transition T in form (2), it is assumed that a change in the logical structure can take place starting with $n = 1$, i.e., from the second cycle. In order to consider the possibility of changes in the structure from the first cycle, the matrix of probabilities of transition must be modified in the following way:

	M_0	M_1	M_2	M_3	M_{12}
M_0	$(1-\delta_0)Q^{(0)}$	$a_1Q^{(1)}$	$a_2Q^{(2)}$	$a_3Q^{(3)}$	0
M_1	$b_1Q^{(0)}$	$(1-\delta_1)Q^{(1)}$	0	0	$a_2Q^{(12)}$
M_2	$b_2Q^{(0)}$	0	$(1-\delta_2)Q^{(2)}$	0	$a_1Q^{(12)}$
M_3	$b_3Q^{(0)}$	0	0	$(1-\delta_3)Q^{(3)}$	0
$T = M_{12}$	0	$b_2Q^{(1)}$	$b_1Q^{(2)}$	0	$(1-\delta_{12})Q^{(12)} \rightarrow$
M_{12}	0	$b_2Q^{(1)}$	0	$b_1Q^{(2)}$	0
M_{23}	0	0	$b_2Q^{(2)}$	$b_2Q^{(3)}$	0
M_{123}	0	0	0	0	$b_3Q^{(12)}$
$M_{\text{отн}}$	0	0	0	0	0

	M_{12}	M_{23}	M_{123}	$M_{\text{отн}}$
	0	0	0	0
	$a_3Q^{(12)}$	0	0	D'_1
	0	$a_3Q^{(23)}$	0	D'_2
	$a_1Q^{(12)}$	$a_2Q^{(23)}$	0	D'_3
\rightarrow	0	0	$a_3Q^{(123)}$	D'_{12}
	$(1-\delta_{12})Q^{(12)}$	0	$a_2Q^{(123)}$	D'_{13}
	0	$(1-\delta_{23})Q^{(23)}$	$a_1Q^{(123)}$	D'_{23}
	$b_2Q^{(12)}$	$b_1Q^{(23)}$	$(1-\delta_{123})Q^{(123)}$	D'_{123}
	0	0	0	1

(7)

Using the properties of a discrete transform, let us find the average number of cycles of proper work of the finite automaton

$$n_{cp} = P(z)|_{z=1}, \quad (8)$$

where $P(z)$ - the depiction of the function of reliability $P(n)$.

For those of the considered (during the calculation of reliability) disturbances, which cannot be removed or which bear a stable character and do not possess the ability of self-restoration during work, one should make $b_1 = 0$.

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